Multilevel Cross-Classified and Multi-Membership Models

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What are Cross-Classified Data?

- Subjects are classified by two or more types of clusters, but clusters are not hierarchical or nested within one another

data collected on	*who are clustered within*
students	schools and neighborhoods
patients	providers and hospitals

Here,

- students from the same neighborhoods go to different schools
- patients can be seen by the same provider at different hospitals

Nested or Hierarchical Structure

	School 1	School 2	School 3	School 4
Neighborhood 1	x x x	x x		
Neighborhood 2			x x x x	x x x x x

students nested within schools within neighborhoods

Crossed Structure

	School 1	School 2	School 3	School 4
Neighborhood 1	x x x x	x x	x x x	x x
Neighborhood 2	x x x		x x	x x x x

students nested within crossing of schools and neighborhoods

$\Rightarrow$ interest is on assessing effects of schools and neighborhoods on student outcomes

Example: Leckie, G. (2013). Cross-Classified Multilevel Models. LEMMA VLE Module 12. (http://www.bristol.ac.uk/cmm/learning/course.html)

Data from 3,435 children who attended 148 primary schools and 19 secondary schools in Scotland.

- VRQ: A verbal reasoning score from tests pupils took when they entered secondary school
- ATTAIN: Attainment score of pupils at age 16
- PID: Primary school identifying code
- SEXF: Pupil's gender (0 = boy and 1 = girl)
- SC: Pupil's social class scale (continuous score from low to high social class)
- SID: Secondary school identifying code
- FED: Father's education (0,1)
- CHOICE: Choice of secondary school that they attend (1=first choice, ... 4=fourth choice)
- MED: Mother's education (0,1)

Excel file: xwdata_9var.xlsx

	A	B	C	D	E	F	G	H	I
1	VRQ	ATTAIN	PID	SEXF	SC	SID	FED	CHOICE	MED
2	111	10	1	0	0	9	0	1	0
3	74	2	1	1	0	9	0	1	0
4	92	4	1	1	1	9	1	1	0
5	98	4	1	0	0	9	0	1	0
6	101	8	1	0	31	1	0	4	0
7	92	5	1	1	0	9	0	1	0
8	121	10	1	0	0	9	0	1	0
9	90	5	1	1	0	9	1	1	1
10	81	2	1	1	0	9	0	1	0
11	96	4	1	1	0	9	0	1	0
12	70	1	1	0	0	18	0	1	0
13	76	2	1	1	0	1	0	4	0
14	101	4	1	1	31	9	1	1	0
15	112	9	1	0	0	9	0	1	0
16	103	6	1	1	0	1	0	2	1
17	96	3	1	1	31	9	0	1	0
18	81	3	1	0	0	9	0	1	0
19	100	6	1	0	0	9	0	1	1
20	109	8	1	1	20	9	0	1	0
21	107	9	1	1	0	9	1	1	0
22	102	10	1	0	20	9	0	1	0
23	83	6	1	1	0	1	0	1	0
24	71	2	1	1	0	1	0	1	0
25	105	4	1	1	0	9	0	1	0
26	96	4	1	1	31	9	1	1	0
27	75	2	1	1	0	9	0	1	0
28	81	2	1	0	0	9	0	1	0
29	91	4	1	0	0	9	1	1	0
30	89	6	1	0	0	1	0	1	1
31	83	4	1	1	0	1	0	2	0

data are sorted by Primary School ID (PID), but appear unsorted in terms of Secondary School ID (SID)

SAS code

```
PROC IMPORT OUT= WORK.xw DATAFILE="xwdata_9var.xlsx"
            DBMS=EXCEL;  GETNAMES=YES;
RUN;

PROC MEANS DATA=xw;
RUN;
```

```
                                  The MEANS Procedure

 Variable    Label        N            Mean         Std Dev         Minimum         Maximum
 ---------------------------------------------------------------------------------------------
 VRQ         VRQ       3435      97.8043668      13.2929071      70.0000000     140.0000000
 ATTAIN      ATTAIN    3435       5.6786026       3.0585043       1.0000000      10.0000000
 PID         PID       3435      70.7377001      45.0257180       1.0000000     148.0000000
 SEXF        SEXF      3435       0.4937409       0.5000336               0       1.0000000
 SC          SC        3435       6.8448326      10.8876068               0      31.0000000
 SID         SID       3435      10.2195051       5.5569402       1.0000000      19.0000000
 FED         FED       3435       0.2754003       0.4467808               0       1.0000000
 CHOICE      CHOICE    3435       1.1953421       0.6502375       1.0000000       4.0000000
 MED         MED       3435       0.3420670       0.4744710               0       1.0000000
 ---------------------------------------------------------------------------------------------
```

PROC MIXED analyses (using default REML estimation)

Null model

```
PROC MIXED  DATA=xw COVTEST;
  CLASS pid sid ;
  MODEL attain =  / S;
  RANDOM INT / SUB=pid;
  RANDOM INT / SUB=sid;
RUN;
```

Model including verbal reasoning score as a covariate

```
PROC MIXED  DATA=xw COVTEST;
  CLASS pid sid;
  MODEL attain = vrq / S;
  RANDOM INT / SUB=pid;
  RANDOM INT / SUB=sid;
RUN;
```

SAS abbreviations: S=SOLUTION, SUB=SUBJECT, INT=INTERCEPT

Alternative (simpler) syntax (using default REML estimation)

Null model

```
PROC MIXED  DATA=xw COVTEST;
  CLASS pid sid ;
  MODEL attain =  / S;
  RANDOM pid sid;
RUN;
```

Model including verbal reasoning score as a covariate

```
PROC MIXED  DATA=xw COVTEST;
  CLASS pid sid;
  MODEL attain = vrq / S;
  RANDOM pid sid;
RUN;
```

SAS abbreviation: S=SOLUTION


```
The Mixed Procedure

                 Model Information

Data Set                     WORK.XW
Dependent Variable           ATTAIN
Covariance Structure         Variance Components
Subject Effects              PID, SID
Estimation Method            REML
Residual Variance Method     Profile
Fixed Effects SE Method      Model-Based
Degrees of Freedom Method    Containment


            Class Level Information

Class    Levels    Values

PID         148    1 2 3 4 5 6 7 8 9 10 11 12 13
                   14 15 16 17 18 19 20 21 22 23
                   24 25 26 27 28 29 30 31 32 33
                   34 35 36 37 38 39 40 41 42 43
                   44 45 46 47 48 49 50 51 52 53
                   54 55 56 57 58 59 60 61 62 63
                   64 65 66 67 68 69 70 71 72 73
                   74 75 76 77 78 79 80 81 82 83
                   84 85 86 87 88 89 90 91 92 93
                   94 95 96 97 98 99 100 101 102
```



```
                   103 104 105 106 107 108 109
                   110 111 112 113 114 115 116
                   117 118 119 120 121 122 123
                   124 125 126 127 128 129 130
                   131 132 133 134 135 136 137
                   138 139 140 141 142 143 144
                   145 146 147 148
SID          19    1 2 3 4 5 6 7 8 9 10 11 12 13
                   14 15 16 17 18 19


            Dimensions

Covariance Parameters            3
Columns in X                     1
Columns in Z Per Subject       167
Subjects                         1
Max Obs Per Subject           3435


          Number of Observations

Number of Observations Read           3435
Number of Observations Used           3435
Number of Observations Not Used          0
```


NULL MODEL WITH NO COVARIATES

```
                 Covariance Parameter Estimates

                                   Standard         Z
Cov Parm      Subject    Estimate      Error    Value      Pr > Z

Intercept     PID          1.1300     0.2074     5.45      <.0001
Intercept     SID          0.3722     0.1743     2.14      0.0164
Residual                   8.1107     0.2004    40.46      <.0001


          Fit Statistics

-2 Res Log Likelihood         17150.8
AIC (smaller is better)       17156.8
AICC (smaller is better)      17156.8
BIC (smaller is better)       17150.8


                  Solution for Fixed Effects

                         Standard
Effect       Estimate       Error      DF    t Value    Pr > |t|

Intercept      5.5017      0.1787      18      30.79      <.0001
```


MODEL WITH VERBAL REASONING AS COVARIATE

```
                 Covariance Parameter Estimates

                                   Standard         Z
Cov Parm      Subject    Estimate      Error    Value      Pr > Z

Intercept     PID          0.2747    0.06171     4.45      <.0001
Intercept     SID         0.01436    0.02402     0.60      0.2749
Residual                   4.2546     0.1050    40.54      <.0001


          Fit Statistics

-2 Res Log Likelihood         14859.1
AIC (smaller is better)       14865.1
AICC (smaller is better)      14865.1
BIC (smaller is better)       14859.1


                  Solution for Fixed Effects

                         Standard
Effect       Estimate       Error      DF    t Value    Pr > |t|

Intercept    -10.0257      0.2782      18     -36.04      <.0001
VRQ            0.1600    0.002766    3268      57.87      <.0001
```

SPSS syntax after reading in Excel file (using default REML)

Null Model

```
MIXED
  attain
  /PRINT = SOLUTION TESTCOV
  /RANDOM INTERCEPT | SUBJECT(pid)
  /RANDOM INTERCEPT | SUBJECT(sid) .
```

Model with `vrq` as covariate

```
MIXED
  attain WITH vrq
  /FIXED = vrq
  /PRINT = SOLUTION TESTCOV
  /RANDOM INTERCEPT | SUBJECT(pid)
  /RANDOM INTERCEPT | SUBJECT(sid) .
```


Alternative (simpler) SPSS syntax

Null Model

```
MIXED
  attain BY pid sid
  /PRINT = SOLUTION TESTCOV
  /RANDOM pid sid .
```

Model with `vrq` as covariate

```
MIXED
  attain WITH vrq BY pid sid
  /FIXED = vrq
  /PRINT = SOLUTION TESTCOV
  /RANDOM pid sid .
```


SPSS output

Mixed Model Analysis

Model Dimension

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	VRQ	1		1	
Random Effects	Intercept	1	Variance Components	1	PID
	Intercept	1	Variance Components	1	SID
Residual				1	
Total		4		5	

a Dependent Variable: ATTAIN.

Information Criteria

-2 Restricted Log Likelihood	14859.140
Akaike's Information Criterion (AIC)	14865.140
Hurvich and Tsai's Criterion (AICC)	14865.147
Bozdogan's Criterion (CAIC)	14886.563
Schwarz's Bayesian Criterion (BIC)	14883.563

The information criteria are displayed in smaller-is-better form.
a Dependent Variable: ATTAIN.

Type III Tests of Fixed Effects

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	2179.567	1298.827	.000
VRQ	1	3356.599	3348.771	.000

a Dependent Variable: ATTAIN.

Estimates of Fixed Effects

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	-10.025716	.278189	2179.567	-36.039	.000	-10.571259	-9.480173
VRQ	.160036	.002766	3356.599	57.869	.000	.154614	.165458

a Dependent Variable: ATTAIN.

Estimates of Covariance Parameters

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		4.254604	.104958	40.536	.000	4.053785	4.465372
Intercept [subject = PID]	Variance	.274656	.061712	4.451	.000	.176821	.426624
Intercept [subject = SID]	Variance	.014365	.024020	.598	.550	.000542	.380736

a Dependent Variable: ATTAIN.

Alternative way to run cross-classified multilevel models

- SAS & SPSS perform cross-classified analyses seamlessly, but not all software can
- Some hierarchical multilevel software programs can be "tricked" into running cross-classified models if they allow
  - 3-level models
  - Equality constraints on variances of random effects
  - Zero covariances of random effects

Alternative way to run cross-classified multilevel models

- Identify cluster level with fewest number of clusters; here, 148 primary schools and 19 secondary schools
- Create indicator variables for the secondary schools, `secs1-secs19` (0/1), which indicate the secondary school that a student belongs to (each student belongs to only one)
- Create a variable `cons` that equals 1 for all observations in the dataset
- At the third level, specify `cons` as the level-3 ID variable, and the 19 indicator variables `secs1-secs19` as random effects with EQUAL variance and zero covariances
- At the second level, specify the primary school ID nested within the level-3 ID (`pid(cons)` in SAS) and specify a random intercept

Secondary School Indicator Variables

ID	secs1	secs2	secs3	secs4	secs5	secs6	secs7	...	secs19
sid = 1	1	0	0	0	0	0	0	...	0
sid = 2	0	1	0	0	0	0	0	...	0
sid = 3	0	0	1	0	0	0	0	...	0
sid = 4	0	0	0	1	0	0	0	...	0
sid = 5	0	0	0	0	1	0	0	...	0
sid = 6	0	0	0	0	0	1	0	...	0
sid = 7	0	0	0	0	0	0	1	...	0
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
sid = 19	0	0	0	0	0	0	0	...	1

⇒ 19 indicator variables for the 19 secondary schools


```
DATA xw2; SET xw;
cons=1;
ARRAY secschool(19)  secs1-secs19;
     DO i = 1 TO 19;
     secschool(i)=0;
         IF sid=i THEN secschool(i)=1;
     END;

PROC MIXED DATA=xw2 COVTEST;
CLASS pid cons;
MODEL attain = / S;
RANDOM INT / SUB=pid(cons);
RANDOM secs1-secs19 / SUB=cons TYPE=TOEP(1);
RUN;
```

`TOEP(1)` is a structure in which all of the random effect variances (for the variables `secs1-secs19`) are equal and uncorrelated


```
The Mixed Procedure

                  Model Information

Data Set                     WORK.ONE
Dependent Variable           ATTAIN
Covariance Structures        Variance Components,
                             Banded Toeplitz
Subject Effects              PID(cons), cons
Estimation Method            REML
Residual Variance Method     Profile
Fixed Effects SE Method      Model-Based
Degrees of Freedom Method    Containment


             Class Level Information

Class    Levels    Values

PID         148    1 2 3 4 5 6 7 8 9 10 11 12 13
                   14 15 16 17 18 19 20 21 22 23
                   24 25 26 27 28 29 30 31 32 33
                   34 35 36 37 38 39 40 41 42 43
                   44 45 46 47 48 49 50 51 52 53
                   54 55 56 57 58 59 60 61 62 63
                   64 65 66 67 68 69 70 71 72 73
                   74 75 76 77 78 79 80 81 82 83
                   84 85 86 87 88 89 90 91 92 93
```



```
                   94 95 96 97 98 99 100 101 102
                   103 104 105 106 107 108 109
                   110 111 112 113 114 115 116
                   117 118 119 120 121 122 123
                   124 125 126 127 128 129 130
                   131 132 133 134 135 136 137
                   138 139 140 141 142 143 144
                   145 146 147 148
cons          1    1


            Dimensions

Covariance Parameters            3
Columns in X                     1
Columns in Z Per Subject       167
Subjects                         1
Max Obs Per Subject           3435


          Number of Observations

Number of Observations Read           3435
Number of Observations Used           3435
Number of Observations Not Used          0
```



```
                   Covariance Parameter Estimates

                                        Standard         Z
Cov Parm      Subject      Estimate       Error      Value      Pr > Z

Intercept     PID(cons)      1.1300      0.2074       5.45      <.0001
Variance      cons           0.3722      0.1743       2.14      0.0164
Residual                     8.1107      0.2004      40.46      <.0001


           Fit Statistics

-2 Res Log Likelihood         17150.8
AIC (smaller is better)       17156.8
AICC (smaller is better)      17156.8
BIC (smaller is better)       17150.8


                   Solution for Fixed Effects

                         Standard
Effect       Estimate       Error      DF    t Value    Pr > |t|

Intercept      5.5017      0.1787     147      30.79      <.0001
```


Stata for cross-classified multilevel

- Stata uses this 3-level approach for cross-classified multilevel models in `xtmixed`
- It creates the school indicator variables, so you don't have to
- Uses ML estimation by default
- Stata is case-sensitive (the variable names were all upper case in this example after reading in the Excel file)

```
. import excel "C:\mixdemo\xwdata_9var.xlsx", sheet("Sheet1") firstrow

. xtmixed ATTAIN || _all: R.SID || PID:, reml
```

```
Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log restricted-likelihood = -8575.3795
Iteration 1:   log restricted-likelihood = -8575.3795

Computing standard errors:

Mixed-effects REML regression                   Number of obs      =      3435

-----------------------------------------------------------
                |   No. of       Observations per Group
 Group Variable |   Groups    Minimum    Average    Maximum
----------------+------------------------------------------
           _all |        1       3435     3435.0       3435
            PID |      148          1       23.2         72
-----------------------------------------------------------

                                                Wald chi2(0)       =         .
Log restricted-likelihood = -8575.3795          Prob > chi2        =         .

------------------------------------------------------------------------------
      ATTAIN |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |   5.501727   .1786809    30.79   0.000     5.151519    5.851935
------------------------------------------------------------------------------
```



```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
                 sd(R.SID)   |   .6101043   .1428677      .3855483     .965449
-----------------------------+------------------------------------------------
PID: Identity                |
                  sd(_cons)  |   1.063027   .0975383        .88806    1.272467
-----------------------------+------------------------------------------------
               sd(Residual)  |   2.847926   .0351921       2.77978    2.917744
------------------------------------------------------------------------------
LR test vs. linear regression:       chi2(2) =   280.57   Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

$\Rightarrow$ Stata estimates SDs (and not variances) for the random effects by default; for variance estimates add the option `variance`

```
. xtmixed ATTAIN || _all: R.SID || PID:, reml variance
```


Intraclass correlations

For the null model (no covariates), Residual variance = 8.1107, Primary school variance = 1.13, Secondary school variance = 0.3722

Students from same primary school, but different secondary schools

$$ICC = \frac{1.13}{8.1107 + 1.13 + 0.3722} = 0.118$$

Students from same secondary school, but different primary schools

$$ICC = \frac{0.3722}{8.1107 + 1.13 + 0.3722} = 0.039$$

Students from same primary and secondary schools

$$ICC = \frac{1.13 + 0.3722}{8.1107 + 1.13 + 0.3722} = 0.156$$

Crossed random-effects model with random interaction

- For this, need combinations of primary and secondary schools where there are several observations
- Can relax assumption of additive random effects
- Some secondary schools might benefit students from particular primary schools more than other primary schools

Crossed structure with singletons only - CANNOT estimate interaction

	Secondary 1	Secondary 2	Secondary 3	Secondary 4
Primary 1	x	x	x	
Primary 2	x	x		x
Primary 3	x	x	x	x
Primary 4	x		x	x

Crossed structure with duplications - CAN estimate interaction

	Secondary 1	Secondary 2	Secondary 3	Secondary 4
Primary 1	xxx	xx	xxx	
Primary 2	xx	x		xxx
Primary 3	x	xxx	xx	xx
Primary 4	xxxx		x	xxxx

Crossed random effects with interaction

SAS

```
PROC MIXED COVTEST;
  CLASS pid sid ;
  MODEL attain =  / S;
  RANDOM pid sid pid*sid;
```

SPSS

```
MIXED
  attain BY pid sid
  /PRINT = SOLUTION TESTCOV
  /RANDOM pid sid pid*sid .
```

STATA

```
. xtmixed ATTAIN || _all: R.SID || PID: || SID:, reml variance
```

Intraclass correlations - model with RE interaction

Null model (no covariates), Residual var = 8.0873, Primary var = 0.9144, Secondary var = 0.3376, Interaction var = 0.2335

Students from same primary school, but different secondary schools

$$ICC = \frac{0.9144}{8.0873 + 0.9144 + 0.3376 + 0.2335} = 0.096$$

Students from same secondary school, but different primary schools

$$ICC = \frac{0.3376}{8.0873 + 0.9144 + 0.3376 + 0.2335} = 0.035$$

Students from same primary and secondary schools

$$ICC = \frac{0.9144 + 0.3376 + 0.2335}{8.0873 + 0.9144 + 0.3376 + 0.2335} = 0.155$$

Crossed random effect models - show me the equations!

Subject k nested within crossing of primary schools i and secondary schools j

model with additive random effects

$$y_{ijk} = \boldsymbol{x}'_{ijk}\boldsymbol{\beta} + \upsilon_i + \upsilon_j + \epsilon_{ijk}$$

$$\upsilon_i \sim N(0, \sigma^2_P) \quad \upsilon_j \sim N(0, \sigma^2_S) \quad \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$$

model with random effects interaction

$$y_{ijk} = \boldsymbol{x}'_{ijk}\boldsymbol{\beta} + \upsilon_i + \upsilon_j + \upsilon_{ij} + \epsilon_{ijk}$$

$$\upsilon_i \sim N(0, \sigma^2_P) \quad \upsilon_j \sim N(0, \sigma^2_S) \quad \upsilon_{ij} \sim N(0, \sigma^2_{PS}) \quad \epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$$

Multi Membership Models

- Subjects are nested within more than one cluster

data collected on	*who are clustered within*
students	more than one teacher
patients	more than one provider

Here,

- assume there are known weights that represent the degree of membership for a subject to the different clusters
- sum of weights equals one
- possibly do sensitivity analysis to examine how different choices for weights affect results

Nested or Hierarchical Structure weights

	Teacher 1	Teacher 2	Teacher 3	Teacher 4
Student 1	1	0	0	0
Student 2	0	1	0	0
Student 3	0	0	1	0
Student 4	0	0	0	1

students nested within teachers

Multi Membership Structure weights

	Teacher 1	Teacher 2	Teacher 3	Teacher 4
Student 1	.25	.25	.25	.25
Student 2	0	1	0	0
Student 3	.33	.33	0	.33
Student 4	0	0	.5	.5

students nested within multiple teachers

Example: Leckie, G. (2013). Multiple Membership Multilevel Models. LEMMA VLE Module 13. (http://www.bristol.ac.uk/cmm/learning/course.html)

Simulated data from 1,000 patients who were treated in all by 25 nurses: 400 treated by only one nurse, 300 treated by two nurses, 200 by three nurses, and 100 by four nurses.

- `patient`: Patient ID
- `satis`: Patient post-op satisfaction (mean=0, std=1)
- `assess`: Patient pre-op assessment (mean=0, std=1); higher scores are better
- `nurses`: Number of nurses seen by the patient (1 to 4)
- `n1st`: Nurse ID for patient's 1st nurse
- `n2nd`: Nurse ID for patient's 2nd nurse
- `n3rd`: Nurse ID for patient's 3rd nurse
- `n4th`: Nurse ID for patient's 4th nurse
- `p1`: Proportion of time with nurse 1
- `p2`: Proportion of time with nurse 2
- ⋮ ⋮
- `p25`: Proportion of time with nurse 25
- `h1`: Job Happiness score for nurse 1
- `h2`: Job Happiness score for nurse 2
- ⋮ ⋮
- `h25`: Job Happiness score for nurse 25

Excel file: nursedat2.xlsx - some subjects seen by only 1 nurse

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	patient	satis	assess	nurses	n1st	n2nd	n3rd	n4th	p1	p2	p3	p4	p5	p6
2	1	2.08535	0.779168	1	24	0	0	0	0	0	0	0	0	0
3	2	-0.12707	0.675926	1	10	0	0	0	0	0	0	0	0	0
4	3	-0.33507	-2.2283	1	15	0	0	0	0	0	0	0	0	0
5	4	-0.89718	0.627285	1	16	0	0	0	0	0	0	0	0	0
6	5	1.160396	-0.39504	1	5	0	0	0	0	0	0	0	1	0
7	6	-0.19741	0.219889	1	19	0	0	0	0	0	0	0	0	0
8	7	1.679402	0.93098	1	4	0	0	0	0	0	0	1	0	0
9	8	1.333309	1.040585	1	25	0	0	0	0	0	0	0	0	0
10	9	-1.18201	-0.63619	1	23	0	0	0	0	0	0	0	0	0
11	10	-0.76671	-0.83759	1	5	0	0	0	0	0	0	0	1	0
12	11	0.606306	-0.49204	1	1	0	0	0	1	0	0	0	0	0
13	12	-1.12415	-1.09006	1	12	0	0	0	0	0	0	0	0	0
14	13	1.334182	-1.77805	1	18	0	0	0	0	0	0	0	0	0
15	14	0.292212	0.05709	1	15	0	0	0	0	0	0	0	0	0
16	15	-0.83327	0.118375	1	10	0	0	0	0	0	0	0	0	0
17	16	0.238606	0.787561	1	22	0	0	0	0	0	0	0	0	0
18	17	-0.04273	-0.34001	1	9	0	0	0	0	0	0	0	0	0
19	18	-0.42192	-0.49124	1	25	0	0	0	0	0	0	0	0	0
20	19	-0.41706	0.5773	1	19	0	0	0	0	0	0	0	0	0
21	20	0.96667	0.53848	1	14	0	0	0	0	0	0	0	0	0
22	21	-2.4744	-0.92708	1	8	0	0	0	0	0	0	0	0	0
23	22	-1.06177	-0.42448	1	2	0	0	0	0	1	0	0	0	0
24	23	-1.1788	0.163695	1	21	0	0	0	0	0	0	0	0	0
25	24	0.048997	-0.19239	1	3	0	0	0	0	0	1	0	0	0

value of 1 for only one of the variables `p1` to `p25` (all others equal 0)

Excel file: nursedat2.xlsx - some subjects seen by 2 nurses

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
402	401	-0.58681	0.191419	2	22	16	0	0	0	0	0	0	0	0	0	0
403	402	1.168098	0.177525	2	11	6	0	0	0	0	0	0	0	0.45	0	0
404	403	0.797301	0.335763	2	8	2	0	0	0	0.55	0	0	0	0	0	0.45
405	404	-0.61233	1.781641	2	6	12	0	0	0	0	0	0	0	0.61	0	0
406	405	-0.08651	0.250836	2	21	13	0	0	0	0	0	0	0	0	0	0
407	406	1.249969	-0.70651	2	1	11	0	0	0.74	0	0	0	0	0	0	0
408	407	-1.90045	-0.65104	2	4	22	0	0	0	0	0	0.47	0	0	0	0
409	408	-1.13544	-0.03925	2	12	20	0	0	0	0	0	0	0	0	0	0
410	409	0.402955	0.168182	2	13	9	0	0	0	0	0	0	0	0	0	0
411	410	0.72849	0.266455	2	20	12	0	0	0	0	0	0	0	0	0	0
412	411	1.053603	1.229129	2	10	15	0	0	0	0	0	0	0	0	0	0
413	412	-0.11927	-1.09804	2	13	11	0	0	0	0	0	0	0	0	0	0
414	413	1.613424	1.555686	2	22	4	0	0	0	0	0	0.29	0	0	0	0
415	414	1.050873	1.531523	2	4	21	0	0	0	0	0	0.4	0	0	0	0
416	415	1.30818	0.140375	2	22	5	0	0	0	0	0	0	0.65	0	0	0
417	416	0.117984	1.92247	2	24	25	0	0	0	0	0	0	0	0	0	0
418	417	-1.72695	-1.71542	2	17	1	0	0	0.39	0	0	0	0	0	0	0
419	418	0.460634	1.503006	2	20	22	0	0	0	0	0	0	0	0	0	0
420	419	-1.30291	-1.80717	2	15	23	0	0	0	0	0	0	0	0	0	0
421	420	0.834141	-0.59873	2	15	4	0	0	0	0	0	0.63	0	0	0	0
422	421	-1.75112	1.245672	2	13	6	0	0	0	0	0	0	0	0.56	0	0
423	422	1.034211	1.46297	2	3	17	0	0	0	0	0.74	0	0	0	0	0
424	423	1.118368	-0.40199	2	18	6	0	0	0	0	0	0	0	0.56	0	0
425	424	0.573003	-0.20438	2	13	10	0	0	0	0	0	0	0	0	0	0
426	425	1.315172	0.107181	2	5	14	0	0	0	0	0	0	0.27	0	0	0

Two of the variables `p1` to `p25` are non-zero and their sum equals 1

Similarly, for patients seen by 3 (or 4) nurses: three (or four) of the variables `p1` to `p25` are non-zero and their sum equals 1

Excel file: nursedat2.xlsx - Job Happiness values (mean=0, sd=1) of nurses

AH	AI	AJ	AK	AL	AM	AN	AO	AP	AQ
h1	h2	h3	h4	h5	h6	h7	h8	h9	h10
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006
1.767854	-0.00445	-0.03987	-0.3772	-1.3821	0.922799	-0.11437	-1.30807	0.786372	1.034006

Nurse-level covariate doesn't change value within a column (same nurse), only across columns `h1` to `h25`


```
PROC IMPORT OUT=WORK.nursedat2 DATAFILE="C:\temp\nursedat2.xlsx"
            DBMS=EXCEL;
            GETNAMES=YES;
RUN;

PROC MEANS DATA=nursedat2;
RUN;

DATA ndats;
SET nursedat2;
cons = 1;
ARRAY ps(25)  p1-p25;
ARRAY hs(25)  h1-h25;
     happiness = 0;
     DO i = 1 TO 25;
        happiness = happiness + ps(i)*hs(i);
     END;

PROC MEANS DATA=ndats;
RUN;
```


The MEANS Procedure

Variable	Label	N	Mean	Std Dev	Minimum	Maximum
patient	patient	1000	500.5000000	288.8194361	1.0000000	1000.00
satis	satis	1000	-0.0387974	0.9809753	-2.9819665	3.0405710
assess	assess	1000	0.0204101	0.9700845	-2.9099002	3.2279503
nurses	nurses	1000	2.0000000	1.0005004	1.0000000	4.0000000
n1st	n1st	1000	12.8140000	7.0523145	1.0000000	25.0000000
n2nd	n2nd	1000	7.8110000	8.5426141	0	25.0000000
n3rd	n3rd	1000	3.7510000	6.9372274	0	25.0000000
n4th	n4th	1000	1.2640000	4.5328634	0	25.0000000
p1	p1	1000	0.0442800	0.1633541	0	1.0000000
p2	p2	1000	0.0487800	0.1786310	0	1.0000000
p3	p3	1000	0.0348300	0.1418933	0	1.0000000
p4	p4	1000	0.0364100	0.1521412	0	1.0000000
p5	p5	1000	0.0410500	0.1640424	0	1.0000000
p6	p6	1000	0.0386500	0.1468190	0	1.0000000
p7	p7	1000	0.0396500	0.1549309	0	1.0000000
p8	p8	1000	0.0373400	0.1517833	0	1.0000000
p9	p9	1000	0.0429000	0.1720581	0	1.0000000
p10	p10	1000	0.0440600	0.1767704	0	1.0000000
p11	p11	1000	0.0323600	0.1407880	0	1.0000000
p12	p12	1000	0.0497800	0.1763305	0	1.0000000
p13	p13	1000	0.0487400	0.1728985	0	1.0000000
p14	p14	1000	0.0442600	0.1630725	0	1.0000000
p15	p15	1000	0.0434600	0.1710687	0	1.0000000
p16	p16	1000	0.0353200	0.1456603	0	1.0000000

p17	p17	1000	0.0298700	0.1288965	0	1.0000000
p18	p18	1000	0.0405200	0.1626130	0	1.0000000
p19	p19	1000	0.0388300	0.1544260	0	1.0000000
p20	p20	1000	0.0327900	0.1439258	0	1.0000000
p21	p21	1000	0.0469600	0.1701150	0	1.0000000
p22	p22	1000	0.0401100	0.1623559	0	1.0000000
p23	p23	1000	0.0376800	0.1503983	0	1.0000000
p24	p24	1000	0.0357600	0.1463942	0	1.0000000
p25	p25	1000	0.0356100	0.1463354	0	1.0000000
h1	h1	1000	1.7678539	0	1.7678539	1.7678539
h2	h2	1000	-0.0044529	0	-0.0044529	-0.0044529
h3	h3	1000	-0.0398703	0	-0.0398703	-0.0398703
h4	h4	1000	-0.3772039	0	-0.3772039	-0.3772039
h5	h5	1000	-1.3820966	0	-1.3820966	-1.3820966
h6	h6	1000	0.9227986	0	0.9227986	0.9227986
h7	h7	1000	-0.1143707	0	-0.1143707	-0.1143707
h8	h8	1000	-1.3080722	0	-1.3080722	-1.3080722
h9	h9	1000	0.7863722	0	0.7863722	0.7863722
h10	h10	1000	1.0340058	0	1.0340058	1.0340058
h11	h11	1000	1.5630561	0	1.5630561	1.5630561
h12	h12	1000	-1.4213433	0	-1.4213433	-1.4213433
h13	h13	1000	0.5874567	0	0.5874567	0.5874567
h14	h14	1000	-0.3803294	0	-0.3803294	-0.3803294
h15	h15	1000	1.2916487	0	1.2916487	1.2916487
h16	h16	1000	-1.9858233	0	-1.9858233	-1.9858233
h17	h17	1000	1.0496269	0	1.0496269	1.0496269
h18	h18	1000	-1.0102353	0	-1.0102353	-1.0102353
h19	h19	1000	-0.7836885	0	-0.7836885	-0.7836885

h20	h20	1000	1.0973895	0	1.0973895	1.0973895
h21	h21	1000	-0.9833028	0	-0.9833028	-0.9833028
h22	h22	1000	-0.2067423	0	-0.2067423	-0.2067423
h23	h23	1000	-0.6376706	0	-0.6376706	-0.6376706
h24	h24	1000	0.1376409	0	0.1376409	0.1376409
h25	h25	1000	0.2539080	0	0.2539080	0.2539080
cons		1000	1.0000000	0	1.0000000	1.0000000
happiness		1000	-0.0231534	0.8019001	-1.9858233	1.7678539

SAS syntax for multi-membership multilevel models

```
PROC MIXED  DATA=ndats COVTEST METHOD=ML;
  CLASS cons;
  MODEL satis =  / SOLUTION;
  RANDOM p1-p25 / SUBJECT=cons TYPE=TOEPLITZ(1);

PROC MIXED  DATA=ndats COVTEST METHOD=ML;
  CLASS cons;
  MODEL satis = assess / SOLUTION;
  RANDOM p1-p25 / SUBJECT=cons TYPE=TOEPLITZ(1);

PROC MIXED  DATA=ndats COVTEST METHOD=ML;
  CLASS cons;
  MODEL satis = assess happiness / SOLUTION;
  RANDOM p1-p25 / SUBJECT=cons TYPE=TOEPLITZ(1);
RUN;
```


NULL MODEL

```
                    The Mixed Procedure

                     Model Information

  Data Set                       WORK.NDATS
  Dependent Variable             satis
  Covariance Structure           Banded Toeplitz
  Subject Effect                 cons
  Estimation Method              ML
  Residual Variance Method       Profile
  Fixed Effects SE Method        Model-Based
  Degrees of Freedom Method      Containment

                        Dimensions

         Covariance Parameters             2
         Columns in X                      1
         Columns in Z Per Subject         25
         Subjects                          1
         Max Obs Per Subject            1000


                  Number of Observations

     Number of Observations Read           1000
     Number of Observations Used           1000
     Number of Observations Not Used          0
```


Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
Variance	cons	0.2323	0.07503	3.10	0.0010
Residual		0.8145	0.03689	22.08	<.0001

Fit Statistics

-2 Log Likelihood	2686.0
AIC (smaller is better)	2692.0
AICC (smaller is better)	2692.0
BIC (smaller is better)	2686.0

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	-0.02653	0.1006	975	-0.26	0.7921

WITH PATIENT COVARIATE ASSESS

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
Variance	cons	0.2550	0.07893	3.23	0.0006
Residual		0.5875	0.02661	22.08	<.0001

Fit Statistics

-2 Log Likelihood	2368.8
AIC (smaller is better)	2376.8
AICC (smaller is better)	2376.9
BIC (smaller is better)	2368.8

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	-0.03317	0.1039	974	-0.32	0.7497
assess	0.4904	0.02531	974	19.37	<.0001

WITH NURSE COVARIATE JOB HAPPINESS

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
Variance	cons	0.1689	0.05413	3.12	0.0009
Residual		0.5874	0.02660	22.08	<.0001

Fit Statistics

-2 Log Likelihood	2359.4
AIC (smaller is better)	2369.4
AICC (smaller is better)	2369.5
BIC (smaller is better)	2359.4

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	-0.03135	0.08579	974	-0.37	0.7148
assess	0.4912	0.02530	974	19.42	<.0001
happiness	0.2936	0.08710	974	3.37	0.0008

SPSS syntax

```
GET FILE='C:\MixDemo\nursedat2.sav'.

COMPUTE cons=1.

COMPUTE happy = 0.
VECTOR ps = p1 TO p25 / hs = h1 TO h25.
LOOP i = 1 TO 25.
+ COMPUTE happy = happy + ps(i) * hs(i).
END LOOP.

MIXED
  satis WITH assess happy p1 TO p25
  /FIXED = assess happy
  /METHOD = ML
  /PRINT = SOLUTION TESTCOV
  /RANDOM  p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p12
           p13 p14 p15 p16 p17 p18 p19 p20 p21 p22
           p23 p24 p25  | SUBJECT(cons) COVTYPE(ID).
```

SPSS output - Dependent Variable: satis

Model Dimension

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	assess	1		1	
	happy	1		1	
Random Effects	p1 + p2 + ... + p25	25	Identity	1	cons
Residual				1	
Total		28		5	

As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax may yield results that differ from those produced by prior versions. If you are using version 11 syntax, please consult the current syntax reference guide for more information.

Information Criteria

-2 Log Likelihood	2359.406
Akaike's Information Criterion (AIC)	2369.406
Hurvich and Tsai's Criterion (AICC)	2369.467
Bozdogan's Criterion (CAIC)	2398.945
Schwarz's Bayesian Criterion (BIC)	2393.945

The information criteria are displayed in smaller-is-better form.

Type III Tests of Fixed Effects

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	23.106	.134	.718
assess	1	981.019	377.001	.000
happy	1	25.414	11.362	.002

Estimates of Fixed Effects

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	-.031354	.085791	23.106	-.365	.718	-.208781	.146073
assess	.491215	.025299	981.019	19.417	.000	.441569	.540862
happy	.293575	.087095	25.414	3.371	.002	.114348	.472803

Estimates of Covariance Parameters

Parameter	Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Residual	.587433	.026604	22.081	.000	.537538	.641959
p1 + p2 + ... + p25 [subject = cons] Variance	.168940	.054127	3.121	.002	.090160	.316558

Stata syntax

```
. import excel "C:\mixdemo\nursedat2.xlsx", sheet("Sheet1") firstrow

. forvalues j = 1/25 {
  2.        generate p`j'Xh`j' = p`j'*h`j'
  3. }

. egen happiness = rsum(p1Xh1-p25Xh25)

. xtmixed satis assess happiness || _all: p1-p25, nocons covariance(identity)
                                     mle variance
```



```
Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:   log likelihood = -1179.7032
Iteration 1:   log likelihood = -1179.7032

Computing standard errors:

Mixed-effects ML regression                     Number of obs      =      1000
Group variable: _all                            Number of groups   =         1

                                                Obs per group: min =      1000
                                                               avg =    1000.0
                                                               max =      1000


                                                Wald chi2(2)       =    386.22
Log likelihood = -1179.7032                     Prob > chi2        =    0.0000

------------------------------------------------------------------------------
       satis |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      assess |   .4912155   .0252988    19.42   0.000     .4416306    .5408003
   happiness |   .2935754   .0870952     3.37   0.001     .1228719    .4642789
       _cons |  -.0313536   .0857909    -0.37   0.715    -.1995006    .1367934
------------------------------------------------------------------------------
```



```
------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
_all: Identity               |
             var(p1..p25)(1) |   .1689407   .0541275      .0901602    .3165583
-----------------------------+------------------------------------------------
               var(Residual) |   .5874327   .0266036      .5375377    .6419591
------------------------------------------------------------------------------
LR test vs. linear regression: chibar2(01) =   121.39 Prob >= chibar2 = 0.0000
(1) p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p12 p13 p14 p15 p16 p17 p18 p19 p20 p21 p22 p23 p24 p25
```


Multi-Membership Multilevel Model

Patient j nested within nurse(s) i ($i = 1, \ldots, N$)

Model with only patient covariates

$$y_j = \boldsymbol{x}'_j \boldsymbol{\beta} + \sum_{i=1}^{N} p_{ij} \upsilon_i \ + \epsilon_{ij}$$

$$\upsilon_i \sim N(0, \sigma^2_\upsilon) \qquad \epsilon_{ij} \sim N(0, \sigma^2_\epsilon) \qquad \sum_{i=1}^{N} p_{ij} = 1 \ \forall \ j$$

Model with a nurse covariate x_i

$$y_j = \boldsymbol{x}'_j \boldsymbol{\beta} + \left(\sum_{i=1}^{N} p_{ij} x_i \right) \beta_{x_i} \ + \ \sum_{i=1}^{N} p_{ij} \upsilon_i \ + \epsilon_{ij}$$

Model with $N \times p$ nurse covariate matrix $\boldsymbol{X}$ and $N \times 1$ nurse weight vector $\boldsymbol{p}_j$ for patient j

$$y_j = \boldsymbol{x}'_j \boldsymbol{\beta} + \ \boldsymbol{p}'_j \boldsymbol{X} \boldsymbol{\beta}_X \ + \ \boldsymbol{p}'_j \upsilon_i \ + \epsilon_{ij}$$

Intraclass correlation - Leckie (2013)

- pairwise correlation of patients DV within a given cluster (conditional on covariates)
- ICCs vary depending on patient weights

Correlation for two patients cared for by the same nurse for the entire hospital stay (null model: nurse var = .2323, error var = .8145)

$$ICC \;=\; \frac{\sigma_\upsilon^2}{\sigma_\upsilon^2 + \sigma_\epsilon^2} \;=\; \frac{.2323}{.2323 + .8145} = 0.222$$

Correlation for two patients j and j' cared for by the same nurse, say nurse 2, for half their stays ($p_{2j} = p_{2j'} = .5$), but different nurses for the rest of their stays, say $p_{5j} = .5$ and $p_{8j'} = .5$

$$ICC \;=\; \frac{p_{2j}p_{2j'}\sigma_\upsilon^2}{\sqrt{\left(p_{2j}^2 + p_{5j}^2\right)\sigma_\upsilon^2 + \sigma_\epsilon^2}\;\sqrt{\left(p_{2j'}^2 + p_{8j'}^2\right)\sigma_\upsilon^2 + \sigma_\epsilon^2}} \;=\; \frac{.25 \times .2323}{.5 \times .2323 + .8145} = 0.062$$

Summary

- Cross-classified models are useful when subjects are classified in two or more cluster types, but cluster types are not nested within each other; more than 1 level of (crossed) clustering
  - Students within primary and secondary schools

- Multi-membership models apply when subjects are clustered within potentially multiple clusters (at the same level)
  - Patients seen by multiple nurses

- Can be computationally demanding because of specification of many random effects; SAS HPMIXED is potentially useful for this